# Trajectory tracking controller using data fusion for mobile robots 

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#### Abstract

The main method of localization by dead reckoning used in indoor mobile robotics is odometry. This technique is prone to drift and errors accumulate during iteration. If we consider, in addition, wheel slipping on the ground, approximation on tyre pressures and the uneven ground, we can feel the shortcomings of this method. In this paper, a real-time controller for a wheeled mobile robot moving in an incompletely known environment is presented. The system uses odometry to estimate continually the vehicle position, which is correcting by a data fusion through an extended Kalman filter, using an embarked CCD camera. Two sliding mode controllers of the actuators linked with the two rear wheels are inserted in an inner control loop. A robust control of the robot is achieved using this strategy. The navigation control of the robot is made in order that the robot follows a pre-defined trajectory, taken as a reference in an outer control loop, which is only achieved if an accurate absolute localization is made. Simulation works show that the proposed method gives good results.


Keywords- Mobile robot, data fusion, trajectory tracking, extended Kalman filter, real-time control.

## I. INTRODUCTION

This paper proposes a method for position estimation of a mobile robot evolving in a partially known environment. Our approach requires the installation of beacons [1] in the evolution environment of the robot to determine its position correctly. The suggested system uses odometry [2] to estimate the vehicle position continuously and corrects the latter at regular intervals by identifying some beacons installed in the work environment using a CCD camera [3], [4]. The camera turns around an axis attached to the robot and measures the azimuth angles of these beacons whose position is known in the evolution environment of the mobile robot (figure 2). Consequently it is the aim of our present paper to show the importance of a precise localization [5]. An extended Kalman filter, answering to the non-linearity of the state and measurements equations of our system, is applied to estimate the optimal values of the position and the orientation by a data fusion approach [6]. A mobile robot must be able to locate
itself but also to work out strategies of motion in order to minimize traveling time, consumed energy, etc... . Moreover during the execution of a task, the robot is brought either to continue a pre-defined trajectory [7], [8] in an absolute reference system, or to avoid obstacles detected on its way, or to follow a path [9], etc... Various sensors collect the localization data and the proposed controller synthesizes adequate control signals in real time, which drive the actuators (servo-motors). The used adaptive control in inner control loops, in conjunction of an effective LQR controller which allows, with the incorporation of the extended Kalman filter in an outer control loop, the correct motion of the mobile robot are presented in this work. Simulation works concerning the mobile robot localization and trajectory tracking are presented. Finally some results of the real-time simulation of the navigation with the presence of different disturbances are reviewed.

## II. PRESENTATION OF THE REAL SYSTEM AND MEASUREMENTS EQUATIONS

By using the laws of traditional mechanics, one can clarify a mobile robot evolution by a state equation (1). The data from CCD sensor give information on this state and constitute the observation equation (2).

$$
\begin{align*}
\dot{x}(t) & =F(x(t), u(t))  \tag{1}\\
\lambda(t) & =G(x(t), u(t)) \tag{2}
\end{align*}
$$

In this expressions $t$ is time, $x(t) \in \mathfrak{R}^{\mathrm{n}}$ is the state vector, $\lambda(t) \in \Re^{\mathrm{m}}$ is the measurements vector and $u(t)$ is the entry. The simplest kinematic model corresponding to a robot moving on a ground flat and driven via indeformable wheels is given by

$$
\left(\begin{array}{c}
\dot{x}  \tag{3}\\
\dot{y} \\
\dot{\psi}
\end{array}\right)=\left(\begin{array}{ccc}
\cos & \psi & 0 \\
\sin \psi & 0 \\
0 & 1
\end{array}\right) \cdot\binom{v}{\omega}
$$

In this expression $v$ is the translation speed of the characteristic point C and $\omega$ is the rotational speed. $U=\left[\begin{array}{ll}v & \omega\end{array}\right]^{T}$ is the entry of the kinematic model (3) and $X=\left[\begin{array}{lll}x & y & \psi\end{array}\right]^{T}$ represents the state.

$\mathbf{X}_{\mathbf{c}}$
Figure 1. Vehicle configuration.
The point C represents the robot gravity center and the medium point of the wheels axis. R is the wheels ray and E indicates the vehicle way. The angle $\psi$ represents the vehicle orientation. The model presented above is based on very reducing simplifying assumptions (rigid wheels, no slip..), in general, one takes account of this approximate aspect of the model through a state noise $V$ which one adds to the state equation (1). Like the treatment and inspecting devices are numerical, the evolution equation, for example the equation (3) is generally presented in discrete time as in (5) where $X(n)$ $=X\left(t_{n}\right)$ and $V_{n}$ is the state noise disturbing the system evolution between the moments $t_{n}$ and $t_{n+l}$.

$$
\left[\begin{array}{l}
x_{n+1} \\
y_{n+1} \\
\psi_{n+1}
\end{array}\right]=\left[\begin{array}{l}
x_{n} \\
y_{n} \\
\psi_{n}
\end{array}\right]+\left[\begin{array}{cc}
\cos \left(\psi_{n}+\frac{\omega_{n .( }\left(t_{n+1}-t_{n}\right)}{2}\right) & 0 \\
\sin \left(\psi_{n}+\frac{\omega_{n \cdot( }\left(t_{n+1}-t_{n}\right)}{2}\right) & 0 \\
0 & \left(t_{n+1}-t_{n}\right)
\end{array}\right] \cdot\left[\begin{array}{l}
v_{n} \\
w_{n}
\end{array}\right]+V_{n}(4)
$$

If one places oneself at one particular moment noted $t_{n}$ all measurements are not available; thus the dimension of the vector observation (6) depends on time.

$$
\begin{equation*}
\lambda_{n}=G_{n}\left(X_{n}, U_{n}\right) \tag{5}
\end{equation*}
$$

One takes into account the inaccuracy related to the process of measurement through an additive noise $W_{n}$, called measurement noise (7).

$$
\begin{equation*}
\lambda_{n}=G_{n}\left(X_{n}, U_{n}\right)+W_{n} \tag{6}
\end{equation*}
$$

The localization problem arises in these terms: at the moment $t_{n}$ we seek an estimate of the robot state starting from the knowledge of the entries $\left(U_{0}, U_{1}, \ldots, U_{n-1}\right)$ applied to the
robot and of measurements $\left(\lambda_{0}, \lambda_{1}, \ldots, \lambda_{n-1}\right)$ available until the moment $t_{n}$ as an environment model if one uses external measurements. Let us note that the entries can be obtained from the control system or from odometers. The state noise $V_{n}$ then takes account of the entry precision.

The observations are the angles of azimuth $\lambda_{i}$ of the beacons $\mathrm{b}_{\mathrm{i}}$ whose coordinates are ( $x i, y i$ ). The equation of observation or measurement makes it possible to connect the current configuration of the robot to the observation sensor. The latter is formulated in the form of angles between the beacons directions and the robot symmetry axis such as:

$$
\begin{equation*}
\lambda_{i}=\operatorname{atg}\left(y_{i}-y / x_{i}-x\right)-\psi=g_{i}(x) \tag{7}
\end{equation*}
$$

$\lambda_{i}$ are the angles which the beacons with the symmetry axis of the mobile robot form and $i$ indicates the beacons number in the environment.

One recognizes by this formulation the no linearity of the state model like that of the measurements model, which leads us to use the extended Kalman filter. The system solution consists in linearizing the equations of the state model as well as measurement model around the estimated vector by using Jacobien of the state and measurement models.


Figure 2. Azimuth angles representation.

## III. ESTIMATE OF THE REAL POSITION

By reference on figure 1, one notes the robot position and orientation at the stage $k$ by the state vector $X(k)=[x(k) y(k)$ $\psi(k) J^{T}$ including a Cartesian position and an orientation laid down compared to a total reference mark. With initialization the robot starts of a known position and has a priori a plan of situation of the beacons $b_{i}$. Each beacon is supposed to be precisely known. With each stop, observations $\lambda_{j}(k)$ of these beacons are taken. Our goal in this cyclic process is to associate measurements $\lambda_{j}(k)$ with the correct beacon $\mathrm{b}_{\mathrm{i}}$ to calculate a new estimate of the vehicle position.

The extended Kalman filter [10] uses the two preceding models: the state model and the measurements model. The state model describes how the robot position $X(k)$ changes
with the response time to an entry of control $u(k)$. It is supposed to be disturbed by a centered Gaussian white vibration v , characterized by its covariance matrix $Q(k)$, then the state equation is written:

$$
\begin{equation*}
X(k+1)=F(X(k), u(k))+v(k) \tag{8}
\end{equation*}
$$

With $v(k) \sim N(0, Q(k))$ and $F(X(k), u(k))$ is a non-linear transition function from state.

The measurements model expresses the sensors observations, according to the vehicle position and the azimuth angles of the detected beacons, with the following form:

$$
\begin{equation*}
\lambda(k)=g_{\mathrm{i}}\left(b_{i}, X(k)\right)+w(k) \tag{9}
\end{equation*}
$$

With $w(k) \sim N\left(0, R_{j}(k)\right)$
The observations function $g_{i}\left(b_{i}, \quad X(k)\right)$ express a measurement observed $\lambda(k)$ like a function of the vehicle position $X(k)$ and of the beacon position. This observation is disturbed by a Gaussian and centered white vibration $w$, characterized by its covariance matrix $R(k)$.

In the case of our application where the equations of state and observation are nonlinear, one uses an approximation which consists in considering these equations linearized around the current estimate to be able to apply the Kalman filter equations like a linear system. One speaks then about extended Kalman filter. The simplest formulation of the filter supposes that the noises are centered, white, decorreled between them and of the initial state and known covariance.
The goal of this algorithm or this cyclic calculation is to produce an estimate of the robot position $\hat{x}(k+1 / k+1)$ at the stage $k+l$ based on the position estimate $\hat{x}(k / k)$ at the stage $k$, on the control entry $U(k)$ and on the new beacon observation $\lambda(k+1)$. The algorithm employs the following stages: prediction, observation, put in correspondence and estimate using the following equations:

$$
\begin{gather*}
\hat{x}(k+1 / k+1)=\hat{x}(k+1 / k)+W(k+1) \cdot v(k+1)  \tag{10}\\
P(k+1 / k+1)=P(k+1 / k)-W(k+1) \cdot S(k+1) \cdot W^{T}(k+1)  \tag{11}\\
W(k+1)=P(k+1 / k) \cdot \nabla g^{t} \cdot S^{-1}(k+1) \tag{12}
\end{gather*}
$$

The equations (10), (11) and (12) represent respectively the new estimated position of the vehicle, the covariance matrix of the state estimation error at the moment $\mathrm{k}+1$ and the Kalman gain vector.
$S(k+1)$ is the result of the stacking of the validated observations Gi. This validation is obtained by an adequate procedure of mapping starting from a certain threshold e. These validated measurements must check the inequality:

$$
\begin{equation*}
V_{i}(k+1) \cdot S^{-1}(k+1) \cdot V_{i}^{t}(k+1) \leq e^{2} \tag{13}
\end{equation*}
$$

Measurements not checking this inequality are simply ignored during the process of localization (they are not used for the phase of estimate).
The term $V_{i}(k+1)$ called innovation is defined like the difference between the real observation and the predicted observation.

$$
\begin{equation*}
V_{i}(k+1)=G_{i}(k+1)-\hat{\lambda}_{i}(k) \tag{14}
\end{equation*}
$$

## IV. CONTROLLER STRUCTURE

The principle diagram and the structure of the designed real time controller for the mobile robot navigation under the software Matlab-Simulink is shown in figure 3.


Figure 3. Real time controller structure
$\alpha_{r}, \alpha_{1}$ are the duty cycles for each chopper which controls the right and the left servo-motors linked with the rear robot wheels. Variable structure PI regulators (which gains $k_{p}$ and $k_{i}$ are computed by the sliding mode controllers) generate them.
$\mathrm{w}_{\mathrm{r}}, \mathrm{w}_{1}$ : The actual velocities in radians per second of the wheels of the robot respectively right and left, and $\mathrm{w}_{\mathrm{rr}}, \mathrm{w}_{\mathrm{rl}}$ are the reference rotational speeds of the wheels.
$\mathrm{L}_{\mathrm{r}}, \mathrm{L}_{\mathrm{rr}}$ : Relative localization of the two driving wheels (respectively left and right wheel).
Lrc: Are the co-ordinates of the center of inertia of the robot in real time and Cc : are the co-ordinates of the center of inertia of the system along a trajectory stored in memory.
HLC: The high level controller which uses data fusion in the outer loop (by application of the extended Kalman filter) provided by the different captors and determinates the speeds references $\mathrm{w}_{\mathrm{r} \mathrm{l}}, \mathrm{w}_{\mathrm{rr}}$.
The inner loop of the controller includes the transfer function of the two servomotors.

## v. SIMULATION WORKS

The basis of our simulation program is to study the localization (figures 4 and 5) and the behavior of the mobile robot such that it can follow a planified path, which is shown
in figure (6). Figure (7) represents wheels speed in the presence of a disturbance and figure (8) represents the duty cycles variation for the servomotors.
The robot moves with a constant translational speed of 0.5 $\mathrm{m} / \mathrm{s}$. The sampling period of the rotational speeds equals 1 ms . Like any other recursive method, the Kalman filter needs an initial estimate $\left(\hat{X}_{0} \hat{Y}_{0} \hat{\psi}_{0}\right)^{\prime}$ with its covariance matrix $\mathrm{P}_{0 / 0}$ The robot begins its movement starting from the initial position $\left(x_{0}, y_{0}, \theta_{0}\right)$.
For the quadratic linear regulator, we have set the following matrices $Q_{I}, R_{I}$ in order to have $K$ as follows:

$$
\begin{aligned}
R_{1}=\left[\begin{array}{cc}
\frac{1}{0.0025} & 0 \\
0 & \frac{1}{0.0025}
\end{array}\right] \quad Q_{1}=\left[\begin{array}{ccc}
\frac{1}{0.0025} & 0 & 0 \\
0 & \frac{1}{0.0025} & 0 \\
0 & 0 & \frac{1}{0.0025}
\end{array}\right] \\
K_{\text {Ricatti }}=\left[\begin{array}{ccc}
0.0025 & 0.0025 & 0 \\
0 & 0 & 0.0025
\end{array}\right]
\end{aligned}
$$




Figure 4. a) Trajectories with and without correction - Example 1 b)Comparison of localization errors for example 1



Figure 5. a) Trajectories with and without correction - Example 2 b)Comparison of localization errors for example 2



Figure 6. a) Trajectory tracking without control, b) Trajectory tracking using the proposed controller


Figure 7. a) Left wheel speed in the presence of a disturbance, b) Right wheel speed in the presence of a disturbance


Figure 8. Duty cycles variation respectively for the left and right servomotor

## VI. CONCLUSION

This paper demonstrates how it is possible to locate a mobile robot and to correct its trajectory using a real time controller. A trajectory estimator using internal (encoders) and external (CCD camera) sensors is presented. We have presented the application of this system for updating the position and the heading of the robot with respect to known beacons in an indoor environment. Further beacons can also be added to improve the robustness and accuracy of the model. Kalman filtering techniques are particularly suited to fusing the data of redundant beacons and also the exclusion of
spurious information. Our main interest has been the real time navigation of a mobile robot. To achieve our goal a discreet model of the robot has been made and we have worked out a technique which localizes the robot by multi-sensorial data fusion, and which includes the real time control of the robot actuators. Thus the robot is indirectly controlled in an inner loop. Tests of simulation with various disturbances, such as when the load torque on each right or left wheel of the robot increases or decreases when evolving, have been successfully achieved. These simulation tests show that a satisfactory path following of a specified trajectory for the mobile robot motion even in the presence of the disturbances has been obtained. This simulation work has used various linear and circular reference paths to show the real efficiency of this control strategy. All simulation works undertaken show a convergence of the model suggested. According to the type of disturbances sullying the mobile robot evolution, one notice that the trajectory correction is done normally by using our control model. Figures 4, 5 and 6 represent some courses of the mobile robot, by taking account of the disturbances which can as well influence the robot state as the observations acquired by CCD sensor and which can intervene at any moments (figure 7). One notes each time that the robot is able to repositioned and to follow the reference trajectory with the help of acquisition of new observations and with their fusion through the controller. The performance and validity of the proposed method were evaluated through a series of simulation works under different random disturbances.

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